Quantum Mechanics

$$\tilde{X} = \sqrt{\frac{\hbar}{2m\omega}}(a^\dagger + a)$$
$$\tilde{P} = i\sqrt{\frac{m\hbar\omega}{2}}(a^\dagger - a)$$
$$a|n\rangle = \sqrt{n}|n-1\rangle$$
$$a^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$$
$$J_x = \frac{1}{2}(J_+ + J_-)$$
$$J_y = \frac{1}{2i}(J_+ - J_-)$$

$$J_\pm|j,m\rangle = \hbar\sqrt{j(j+1) - m(m\pm 1)}|j,m\pm 1\rangle$$
$$S_i = \frac{\hbar}{2}\sigma_i$$
$$[S_i, S_j] = i\hbar\epsilon_{ijk}S_k$$

$$< S_z >= \mathrm{Tr}[\rho S_z]$$

$$L^2|l,m\rangle = l(l+1)\hbar^2|l,m\rangle$$
$$L_z|l,m\rangle = m\hbar|l,m\rangle$$

$$H = \frac{1}{2m}(\vec{p} - q\vec{A})^2 + q\phi$$
$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2$$

$$E = \frac{\hbar^2\kappa^2}{2m}$$
$$E_n = \hbar\omega(n + \frac{1}{2})$$

$$W_{fi} = \frac{1}{\hbar^2}|\int_{-\infty}^{\infty}\langle f|H'|i\rangle e^{i(E_f - E_i)t/\hbar}dt|^2$$

$$\omega = \frac{eB}{2m}$$
$$< m >= \int \psi^* m\psi dx$$

$$\Delta m = \sqrt{< m^2 > - < m >^2}$$
$$< p >= \frac{1}{m}\frac{d}{dt} < x >$$

$$\frac{\lambda_1}{\lambda_2} = \frac{\mu_1}{\mu_2}$$
$$e^{i\phi\hat{n}\cdot\vec{S}/\hbar} = I\cos(\phi/2) + i\hat{n}\cdot\vec{\sigma}\sin(\phi/2)$$

$$\sigma_{tot} = \frac{4\pi}{k^2}\sum_l (2l+1)\sin^2(\delta_e(k))$$
$$\frac{\hbar^2}{2m}\frac{\partial^2 U}{\partial r^2} - \frac{l(l+1)\hbar^2}{2mr^2}U + VU = EU$$
$$U = rR(r)$$

$$\text{Spherical Bessel functions}:$$
$$A_l j_l(kr),\ B_l y_l(kr)$$
$$\tan(\delta_l) = -\frac{B_l}{A_l}$$
$$\text{small } k : j_l(kr) \to \frac{1}{kr}\sin(kr)$$
$$y_l(kr) \to -\frac{1}{kr}\cos(kr)$$

$$k\cot\delta_0 \approx -\frac{1}{a}$$
$$kr > \sqrt{l(l+1)}$$
$$\sigma \approx \pi R^2$$
$$f(\theta) = -\frac{m}{2\pi\hbar^2}\int V(r)e^{i\vec{\kappa}\cdot\vec{r}}dr$$
$$\text{spherical}: f(\theta) = -\frac{2m}{\hbar^2\kappa}\int_0^\infty rV(r)\sin(\kappa r)dr$$
$$\text{low energy}: f(\theta) = -\frac{m}{2\pi\hbar^2}\int V(r)dr^3$$
$$\frac{d\theta}{d\Omega} = |f(\theta)|^2$$
$$\kappa = 2k\sin(\frac{\theta}{2})$$

$$E^{(1)} = \langle\phi^{(0)}|H'|\phi^{(0)}\rangle$$
$$|\phi_n^{(1)}\rangle = \sum_{k\neq n}\frac{\langle\phi_k^{(0)}|H'|\phi_n^{(0)}\rangle}{E_n^0 - E_k^0}|\phi_k^{(0)}\rangle$$
$$E_n^{(2)} = \sum_{k\neq n}\frac{|\langle\phi_k^{(0)}|H'|\phi_n^{(0)}\rangle|^2}{E_n^{(0)} - E_k^{(0)}}$$

$$T = e^{-2\gamma}$$
$$\gamma = \frac{1}{\hbar}\int_{x_1}^{x_2}\sqrt{2m[V(x) - E]}dx$$
$$p(x) = \sqrt{2m(E - V)}$$
$$\int_{x_1}^{x_2} p(x)dx = (n - \frac{1}{2})\pi\hbar$$

$$\psi(p) = \frac{1}{2\pi\hbar}e^{\hbar\omega(n+c)}$$
$$\text{one hard wall}: c = \frac{1}{2}$$
$$2\text{ hard walls}: c = 1$$
$$\text{no hard walls}: \text{c} = \frac{3}{4}$$

$$\frac{-\hbar^2}{2\mu}\frac{d^2U}{dr^2} + \frac{l(l+1)\hbar^2}{2\mu r^2}U + V(r)U = EU$$

$$a \propto \frac{1}{\mu e^2}$$

$$E_v = \frac{\langle\psi_{tri}|H|\psi_{tri}\rangle}{\langle\psi_{tri}|\psi_{tri}\rangle}$$

$$\bar{j} = \frac{i\hbar}{2m}[\psi\nabla\psi^* - \psi^*\nabla\psi]$$

Classical Mechanics

$$\mathcal{L} = T - V$$
$$\frac{d}{dt}\frac{\partial\mathcal{L}}{d\dot{\theta}} = \frac{\partial\mathcal{L}}{d\theta}$$
$$H = T + V$$

$$H = -\mathcal{L} + p_i\dot{q}_i$$
$$p_i = \frac{\partial\mathcal{L}}{\partial\dot{q}_i}$$
$$-\dot{p}_i = \frac{\partial H}{\partial q_i};\dot{q}_i = \frac{\partial H}{\partial p_i}$$

$$\frac{\partial H}{\partial t} = \frac{-\partial\mathcal{L}}{\partial t}$$
$$T = \frac{m}{2}(\dot{r}^2 + r^2\dot{\theta}^2 + r^2\sin^2(\theta)\dot{\phi}^2)$$
$$H = \frac{1}{2m}(p_r^2 + \frac{1}{r^2}p_\theta^2 + \frac{1}{r^2\sin^2(\theta)}p_\phi^2)$$
$$H = \frac{1}{2m}(p_r^2 + \frac{1}{r^2}p_\theta^2 + p_z^2)$$

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$
$$v(t) = v_0 + at$$
$$v_f^2 - v_i^2 = 2a(x - x_0)$$
$$x = x_0 + \frac{1}{2}(v_0 + v)t$$
$$\tau = \vec{r} \times \vec{F} = I\vec{\alpha} = \frac{dL}{dt}$$
$$T = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$
$$\omega = \sqrt{\frac{k}{m}}$$
$$f = \frac{\omega}{2\pi}$$
$$L = I\omega$$

$$v_T = 2\pi r f = \omega r$$
$$a_c = \frac{v^2}{r} = -m\vec{\Omega} \times (\vec{\Omega} \times \vec{r})$$
$$F_c = \frac{mv^2}{r}$$
$$F_{\text{cor}} = -2\vec{\Omega} \times \vec{v}$$

$$W = \int \tau d\theta = \vec{F} \cdot \vec{s} = Fs\cos(\theta)$$
$$P = \frac{dW}{dt}$$
$$U = mgh$$
$$U = \frac{1}{2}kx^2$$
$$U = -\frac{Gm_1M_2}{r^2}$$
$$I_{COM} = \int p(r)(\delta_{jk}r^2 - (x_j - x_k))d^3r$$
$$I_{jk} = \sum_i m_i(\delta_{jk}r_i^2 - (x_jx_k)_i)$$
$$I = I_{COM} + md^2$$
$$v = v_i + v_{\text{exhaust}} \ln(\frac{\mu_E}{M})$$
$$F = v_{\text{exhaust}}|\frac{\Delta\mu}{\Delta t}|$$
$$m = m_0 e^{-\frac{v}{u}}$$
$$T = \frac{1}{n}V = \frac{1}{2}V$$

$$V_{\text{eff}} = kr^s + \frac{l^2}{2mr^2}$$
$$l = mR^2\dot{\theta}$$
$$T^2 \propto a^3$$
$$e = \sqrt{1 + \frac{2El^2}{mk^2}}$$
$$M[\begin{matrix} 0 & 1 \\ -1 & 0 \end{matrix}]M^T = [\begin{matrix} 0 & 1 \\ -1 & 0 \end{matrix}]$$
$$M = [\begin{matrix} \frac{\partial Q}{\partial q} & \frac{\partial Q}{\partial p} \\ \frac{\partial P}{\partial q} & \frac{\partial P}{\partial p} \end{matrix}]$$
$$P = \frac{-\partial F}{\partial Q}; q = \frac{-\partial F}{\partial p}$$
$$\vec{\nabla}^2\Phi = 4\pi\rho G$$
$$\int \vec{\nabla}^2\Phi \cdot d\vec{a} = 4\pi\rho G\Sigma_k$$
$$D^2 + bD + \omega_0^2 = 0$$
$$x(t) = e^{ipt}[C\cos(qt) + E\sin(qt)]$$

Statistical Mechanics

$$E = k_BT = \frac{\hbar^2k^2}{2m} = \frac{p^2}{2m}$$
$$I(f) = \frac{C}{4V}E(f)$$
$$\Omega = \frac{(q + N - 1)!}{q!(N-1)!}$$

$$S = k_B\ln(\Omega)$$
$$T = \left(\frac{\partial S}{\partial E}\right)^{-1}$$
$$C_P = T\left(\frac{\partial S}{\partial T}\right)_P = C_V + \frac{TV\alpha^2}{\kappa_T}$$
$$C_V = T\left(\frac{\partial S}{\partial T}\right)_V = (\frac{\partial < E >}{\partial T})_{VN}$$
$$k_T = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)$$
$$dS = \left(\frac{\partial S}{\partial T}\right)_{VN}dT + \left(\frac{\partial S}{\partial V}\right)_{TN}dV + \left(\frac{\partial S}{\partial N}\right)_{TV}dN$$
$$\text{Helmholtz} : F = -k_BT\ln Z$$
$$\text{Gibbs} : G = E - TS + PV$$
$$dE = TdS - PdV + \mu dN$$
$$dV = \left(\frac{\partial V}{\partial T}\right)_P dT + \left(\frac{\partial V}{\partial P}\right)_T dP$$

$$P(v)dv = n(v)v^2\sin(\theta)dvd\theta d\phi$$
$$\bar{v} = \int_0^\infty vPdV$$
$$v^* = \frac{\partial P}{\partial V} = 0$$
$$v_{\text{rms}} = \int_0^\infty v^2PdV$$
$$< n_j(\epsilon) >_{FD} = \frac{1}{e^{\beta(\epsilon-\mu)} + 1}$$
$$< n_j(\epsilon) >_{BE} = \frac{1}{e^{\beta(\epsilon-\mu)} - 1}$$
$$g(\epsilon) = \gamma_S\frac{4\sqrt{2}\pi Vm^{3/2}}{h^3}\epsilon^{1/2}$$
$$N(\tau = 0) = \int_0^{\epsilon_F} g_{FD}(\epsilon)d\epsilon$$
$$< E > = \int_0^\infty \epsilon < n >_{FD} g_{FD}d\epsilon$$
$$\eta = \frac{\omega}{Q_k} \leq 1 - \frac{T_c}{T_H}$$
$$\omega = Q_h - Q_c$$
$$S = \frac{Q}{T}; C = \frac{Q}{\Delta T}$$
$$\frac{dQ_H}{T_H} = -\frac{Q_c}{T_c}$$
$$\Delta T = 0; \Delta S = \frac{Q}{T}; -\omega = Q = \int PdV$$

$$L = \frac{Q}{m}; \Phi = \frac{P}{A}$$
$$P = \sigma AT^4$$
$$\sigma_{rms} = \sqrt{< E^2 > - < E >^2}$$
$$< E > = \frac{f}{2}Nk_BT$$
$$\binom{N}{n}\frac{v}{\nu}^n\left(1 - \frac{v}{\nu}\right)^{N-n}$$
$$\frac{\mu}{\epsilon_F} = 1 - \frac{\pi^2}{12}(\frac{1}{\epsilon_F\beta})^2 + ...$$
$$z_i = \Sigma_je^{-\beta E_j}; Z = \frac{1}{N!}(z_i)^N$$
$$< E > = -\frac{\partial}{\partial\beta}\ln(z)$$
$$< \tilde{O} > = \frac{1}{Z}\Sigma_j\hat{O}_je^{-\beta\epsilon_j}$$
$$T_D = \frac{h}{2k_B}\sqrt{\frac{kT}{\rho}}\left(\frac{6\rho N_A}{\pi M}\right)^{1/3}$$
$$T \lll D : C_V \approx \frac{12}{5}\pi^4Nk_B\left(\frac{T}{T_D}\right)^3$$
$$\Delta S = 0$$
$$C_V = 3R$$

E&M

$$P = \oint(\vec{E} \times \vec{H}) \cdot d\vec{A}$$
$$\vec{L} = \epsilon_0\int \vec{r} \times \vec{E} \times \vec{B}dr^3$$

$$P = \frac{E}{t}$$
$$U_{rad} = \int_0^t Pdt$$

$$\vec{m} = \frac{1}{2}\int \vec{r'} \times \vec{J}(r')d^3r'$$
$$\vec{A} = \frac{\mu_0}{4\pi}\int\frac{\vec{J}(\vec{r'})d^3\vec{r'}}{|\vec{r} - \vec{r'}|}$$

$$\phi = k\int \frac{\rho(\vec{r'})d^3r'}{|\vec{r}-\vec{r'}|}$$

$$\phi = \sum_{lm}[A_{lm}\left(\frac{r}{R}\right)^l + B_{lm}\left(\frac{r}{R}\right)^{-(1+l)}]Y_e^m(\theta,\phi)$$

$$\mathcal{E}_{in}\frac{\partial}{\partial r}\phi_{in}|_{r=R} - \mathcal{E}_{out}\frac{\partial}{\partial r}\phi_{out}|_{r=R} = \sigma$$

$$\vec{J} = \sigma\vec{E}$$

$$\vec{\nabla}\cdot\vec{J} = -\frac{\partial\rho}{\partial t}$$

$$\vec{J} = \vec{\nabla}\times\vec{M}$$

$$\vec{K} = \vec{M}\times\hat{n} = \sigma\vec{v} = \sigma\vec{\omega}\times\vec{r}$$

$$B = \frac{\mu_0}{4\pi}\int\frac{\vec{K}\times(-\vec{r})}{r^3}dS$$

$$\Phi_B = \int\vec{B}\cdot d\vec{A} = IR$$

$$R = \frac{\rho L^2}{V}$$

$$E = E_0e^{i(kz-wt)}$$

$$E_1^{||} = E_2^{||}; \frac{1}{\mu_1}B_1^{||} = \frac{1}{\mu_2}B_2^{||}$$

$$\vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\times\vec{B} = \mu\vec{J} + \mu\epsilon\frac{\partial\vec{E}}{\partial t}$$

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho_{free}}{\epsilon_0}$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$D = \epsilon E; B = \mu H$$

$$\oint\vec{D}\cdot d\vec{S} = Q$$

$$\oint\vec{B}\cdot d\vec{l} = \mu I$$

$$\vec{\nabla}\times\vec{H} \propto (\frac{i\sigma}{\omega} + \epsilon_0\epsilon_r)\frac{\partial E}{\partial t}$$

$$B = \frac{\mu NI}{L}$$

$$V(t) = N\frac{d\Phi}{dt}$$

$$n_1\sin(\theta_1) = n_2\sin(\theta_2)$$

$$I = \frac{P}{A} = \frac{1}{2}v\epsilon|E|^2$$

$$U_C = \frac{1}{2}CV^2$$

$$U_L = \frac{1}{2}LI^2 = \frac{1}{2}\int H\cdot Bdr^3$$

$$\vec{F} = \nabla(\vec{m}\cdot\vec{B})$$

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{3\hat{r}(\hat{r}\cdot\vec{m})-\vec{m}}{r^3}$$

$$\vec{A} = \frac{\mu_0}{4\pi}\frac{\vec{m}\times\vec{r}}{r^3}$$

$$q' = -q\frac{R}{b}; x = \frac{R^2}{b}$$

$$\vec{m}' = \frac{\mu_2-\mu_1}{\mu_2+\mu_1}\vec{m}$$

$$P = \frac{2}{3}\frac{1}{4\pi\epsilon_0}\frac{e^2|a|^2}{c^3}$$

$$P = \frac{2}{3}\frac{1}{4\pi\epsilon_0}\frac{e^2}{c}\gamma^6[\dot{\beta}^2 - (\beta\times\dot{\beta})^2]$$

$$P = \frac{2}{3}\frac{1}{4\pi\epsilon_0}\frac{|\ddot{p}_0|^2}{c^3}$$

$$p_0 = \sum_i q_ir_i$$

$$\begin{pmatrix}ct'\\x'\\y'\\z'\end{pmatrix} = \begin{bmatrix}\gamma & -\beta_x\gamma & 0 & 0\\ -\beta_x\gamma & \gamma & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}\begin{pmatrix}ct\\x\\y\\z\end{pmatrix}$$

$$F = \gamma^3ma_{||} + \gamma ma_\perp$$

$$\tilde{P}(E,D,H,B) = (-E,-D,H,B)$$

$$\tilde{T}(E,D,H,B) = (E,D,-H,-B)$$

General Formulas & Constants

$$\cos^2(u) = \frac{1+\cos(2u)}{2}$$

$$\sin^2(u) = \frac{1-\cos(2u)}{2}$$

$$\sin(a)\sin(b) = \frac{1}{2}(\cos(a-b) - \cos(a+b))$$

$$\cos(\frac{\theta}{2}) = \frac{1}{2}(1+\cos(\theta))$$

$$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$$

$$\Gamma(n) = (n-1)!$$

$$\Gamma(n+\frac{1}{2}) = \sqrt{\pi}\frac{2n!}{4^nn!}$$

$$Q(n) = 1 + \frac{n+3}{n-1}\frac{1}{2^{n+1}}$$

$$R = 8.31J(K*mol)^{-1}$$

$$k_B = 1.381*10^{-23}\frac{J}{K}$$

$$N_A = 6.022*10^{23}\frac{part}{mol}$$

$$amu = 1.661*10^{-27}kg$$

$$\hbar = 6.582*10^{-16}eVs$$

$$eV = 1.602*10^{-19}$$

$$m_e = 0.511\frac{MeV}{c^2}$$

$$m_p = 938\frac{MeV}{c^2}$$

$$a_0 = 5.29*10^{-11}m = \frac{\hbar}{me^2} = \frac{\hbar}{mc\alpha}$$

$$\alpha_{\text{fine}} = \frac{1}{137}$$

$$\epsilon_0 = 8.85*10^{-12}\frac{F}{m}$$

$$\mu_0 = 4\pi*10^{-7}\frac{H}{m}$$

Geometry	Moment of Inertia
Solid Cube	$\frac{1}{6}MR^2$
Solid Cylinder/Disk About Symm. Axis	$\frac{1}{2}MR^2$
Hoop around diameter	$\frac{1}{2}MR^2$
Hoop about symm. axis	MR^2
Solid Sphere	$\frac{2}{5}MR^2$
Rod about center	$\frac{1}{12}ML^2$
Rod about end	$\frac{1}{3}ML^2$
Hollow Sphere	$\frac{2}{3}MR^2$

$$Y_0^0 = \frac{1}{2\sqrt{\pi}}$$

$$Y_1^{-1} = \frac{1}{2}\sqrt{\frac{3}{2\pi}}e^{-i\phi}\sin\theta$$

$$Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta$$

$$Y_1^{+1} = -\frac{1}{2}\sqrt{\frac{3}{2\pi}}e^{i\phi}\sin\theta$$