ERROR IN PROMPT FEEDBACK REACTIVITY AT EBR-II: A PARAMETRIC STUDY

A Thesis

Submitted to the Graduate Faculty of the
Louisiana State University and
Agricultural and Mechanical College
in partial fulfillment of the
requirements for the degree of
Master of Science

in

The Department of Nuclear Engineering

by
Winton George Aubert
B.S., Louisiana State University, 1974
December, 1975

Dedicated to

my parents

ACKNOWLEDGEMENT

The guidance and assistance of Dr. Howard A. Larson of Argonne National Laboratory's EBR-II site is deeply appreciated. Without his encouragement, this thesis would not have been possible. Dr. John I. Sackett and Mr. H. M. "Mac" Forehand, both of Argonne (EBR-II), were also helpful in my endeavor.

I wish to especially thank Dr. John C. Courtney for his role as committee chairman, and Drs. Frank A. Iddings, Robert C. McIlhenny, William F. Curry, and Myron H. Young, all of Louisiana State University, for their suggestions and help in preparing this work. The Argonne Center for Educational Affairs provided the appointments and support for the EBR-II Summer Engineering Institute and the Argonne Thesis Parts Program.

A special thanks is extended to Mrs. Priscilla Milligan for her assistance in the preparation of the thesis manuscript. And finally, I am thankful to my family and friends, for their motivational involvement in this work.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

ABSTRACT

The Experimental Breeder Reactor II (EBR-II), located near Idaho Falls, Idaho, is the major facility of the U. S. Energy Research and Development Administration for the development of liquid metal cooled, fast breeder reactor technology. Among the objectives of EBR-II is the investigation of neutron dynamics in a small, high power core. One specific approach, called inverse kinetics, involves the evaluation of measured system responses by digital computer codes such as RCL and FBCK.

This thesis analyzes suspected errors in feedback reactivity calculated by inverse kinetics codes and compares them to measured data. While the total delayed neutron fraction has essentially no effect on prompt feedback reactivity, significant errors in prompt feedback reactivity may be induced by errors in the individual delayed neutron fractions. The derivative term, dn/dt, of the neutron kinetics equation is examined analytically to determine its importance to fast reactor kinetics analysis. This thesis establishes that the derivative term can be ignored in almost all cases for fast reactors since the prompt neutrons have very short lifetimes. A possible area of further investigation would include an analytic prediction of errors in the feedback reactivity; there is a possibility that such errors could significantly affect the validity of inverse kinetics calculations.

CHAPTER I

Introduction

Of the many different configurations of nuclear fission reactors that are possible through present technology, perhaps the most promising from a fuel economics standpoint is the fast breeder reactor. Breeder reactors are unique in that they utilize nuclear reaction and decay processes to produce more nuclear fuel than they consume as they generate electric power. Although these reactors are perhaps not the final, long-range answer to increasing energy needs, their fuel breeding ability makes them very desirable from a medium-range point of view. A system of electric power generation that includes breeder reactors can greatly extend our fossil and nuclear fuel resources.

At present, there is but one operating fast breeder reactor in the Western Hemisphere, known as Experimental Breeder Reactor II, or EBR-II, located at the Idaho National Engineering Laboratory near Idaho Falls, Idaho, and operated by Argonne National Laboratory's Western branch, ANL-W. EBR-II produces 62.5 megawatts of thermal power and generates about 20 megawatts of electrical power. Originally designed to demonstrate fuel breeding capability, the primary objectives now of EBR-II are fuel irradiation and testing of breeder reactor hardware. This reactor serves as the primary facility of the United States Energy Research and Development Administration (ERDA) for irradiation of fuels in the breeder reactor program.

EBR-II is classified as a liquid metal cooled, fast breeder reactor, or LMFBR. Since the primary coolant is liquid sodium, the nuclear fission reactions are sustained primarily by fast neutrons, and the combination of fuel and neutron energy lead to breeding capability. This is a pool-type reactor, and it employs a stainless steel neutron reflector around the core. The nuclear fuel is uranium which is nominally fifty percent enriched in ^{235}U, with provision for use of fuel with other enrichments. Figure I-1 illustrates the EBR-II reactor vessel and neutron shield configuration, with several important components indicated. The fuel is arranged in a hexagonal array consisting of 637 subassemblies. Each subassembly is a metal clad bundle of fuel pins, with passages for the liquid sodium coolant, and each subassembly can be remotely handled while in the reactor. Most of the fission reactions occur in the driver fuel which is loaded in the core section, with most of the fuel breeding occurring in the inner blanket section. These sections are shown in a plane view of the reactor in Figure I-2. Axial distributions of the fission rates in the driver fuel subassemblies for ^{238}U and ^{239}Pu are shown in Figures I-3 and I-4, respectively.

In order to assure that EBR-II is operated safely and within design limits, reactor kinetics studies are performed periodically. Kinetics studies may be of an experimental nature, performed directly on the reactor, or may use an analytical approach employing computer programs that simulate the dynamic behavior of the reactor. The experimental approach includes analysis of several types of reactor

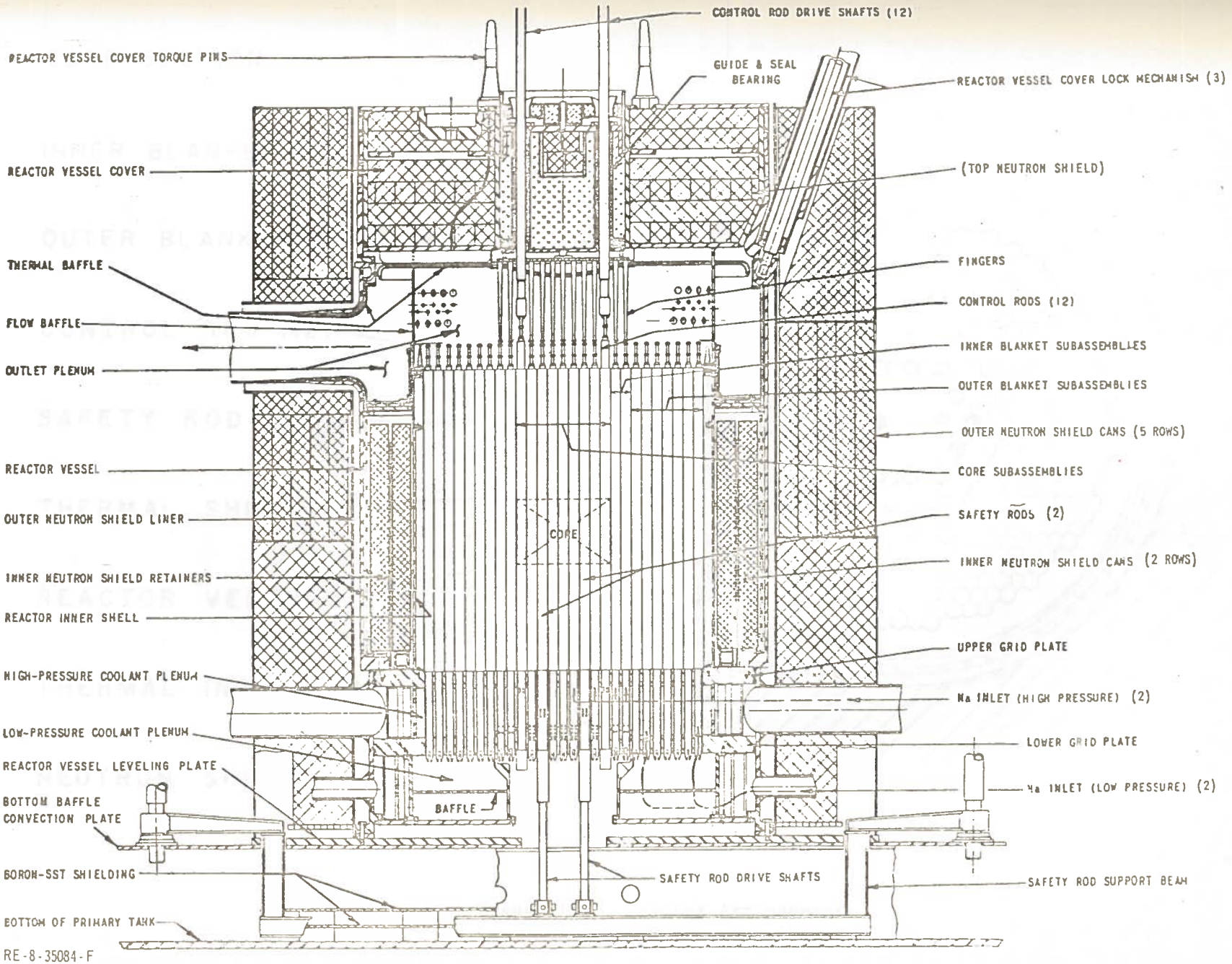

Figure I-1. Reactor Vessel and Neutron Shield Assembly

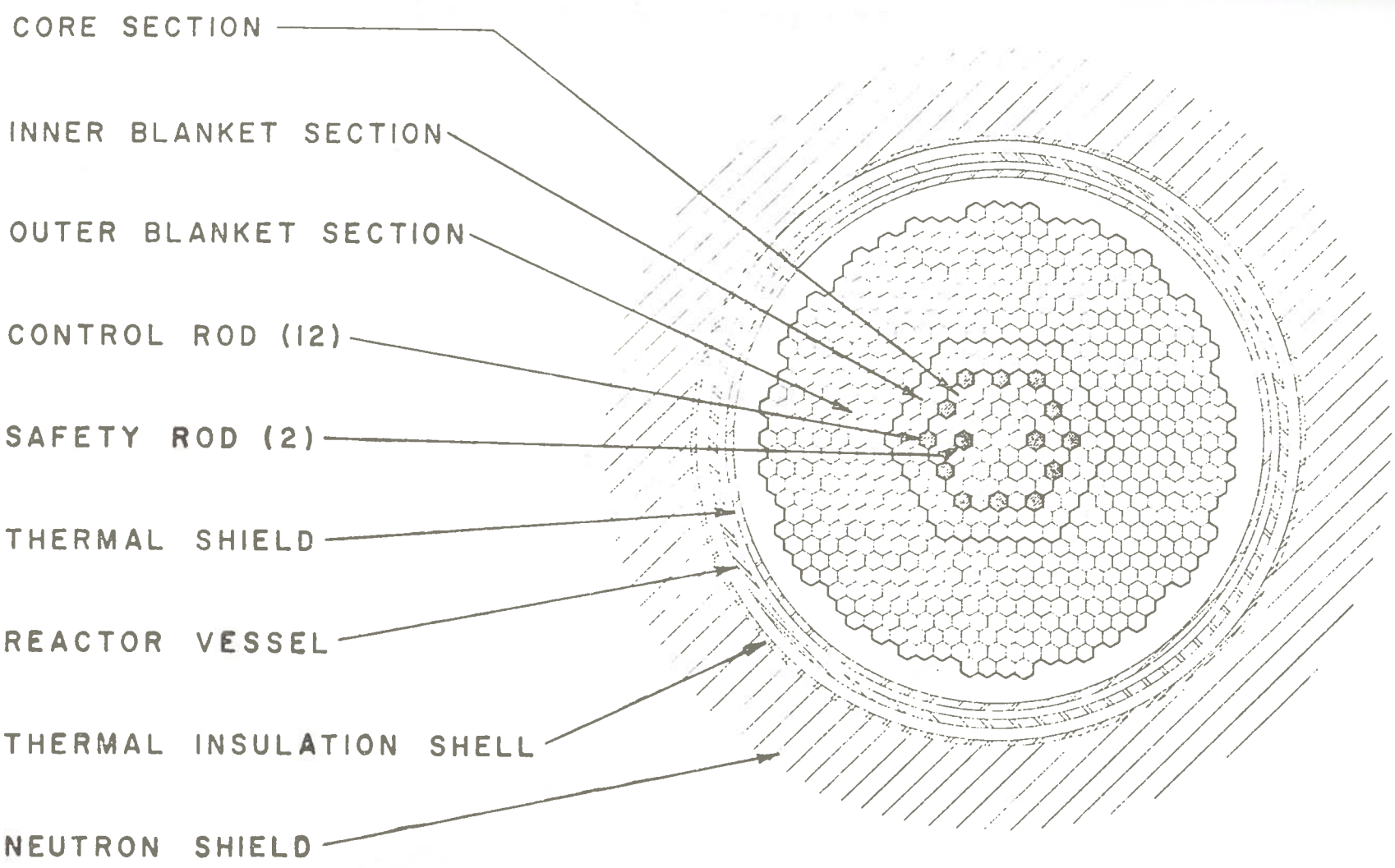

Figure I-2. Reactor Arrangement

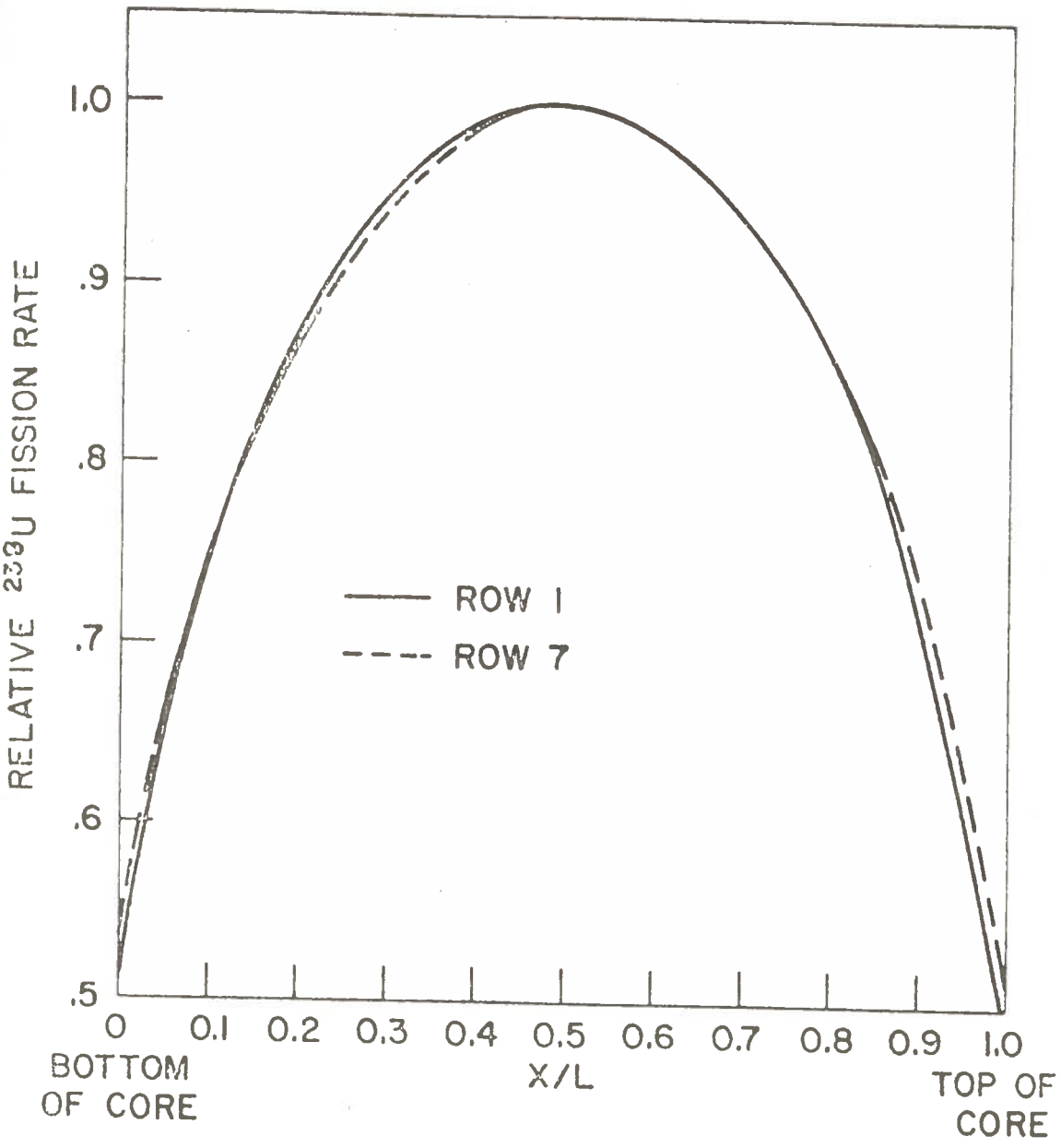

Figure I-3. Axial Distribution of ^{238}U Fission Rate in Driver Fuel in the EBR-II Core

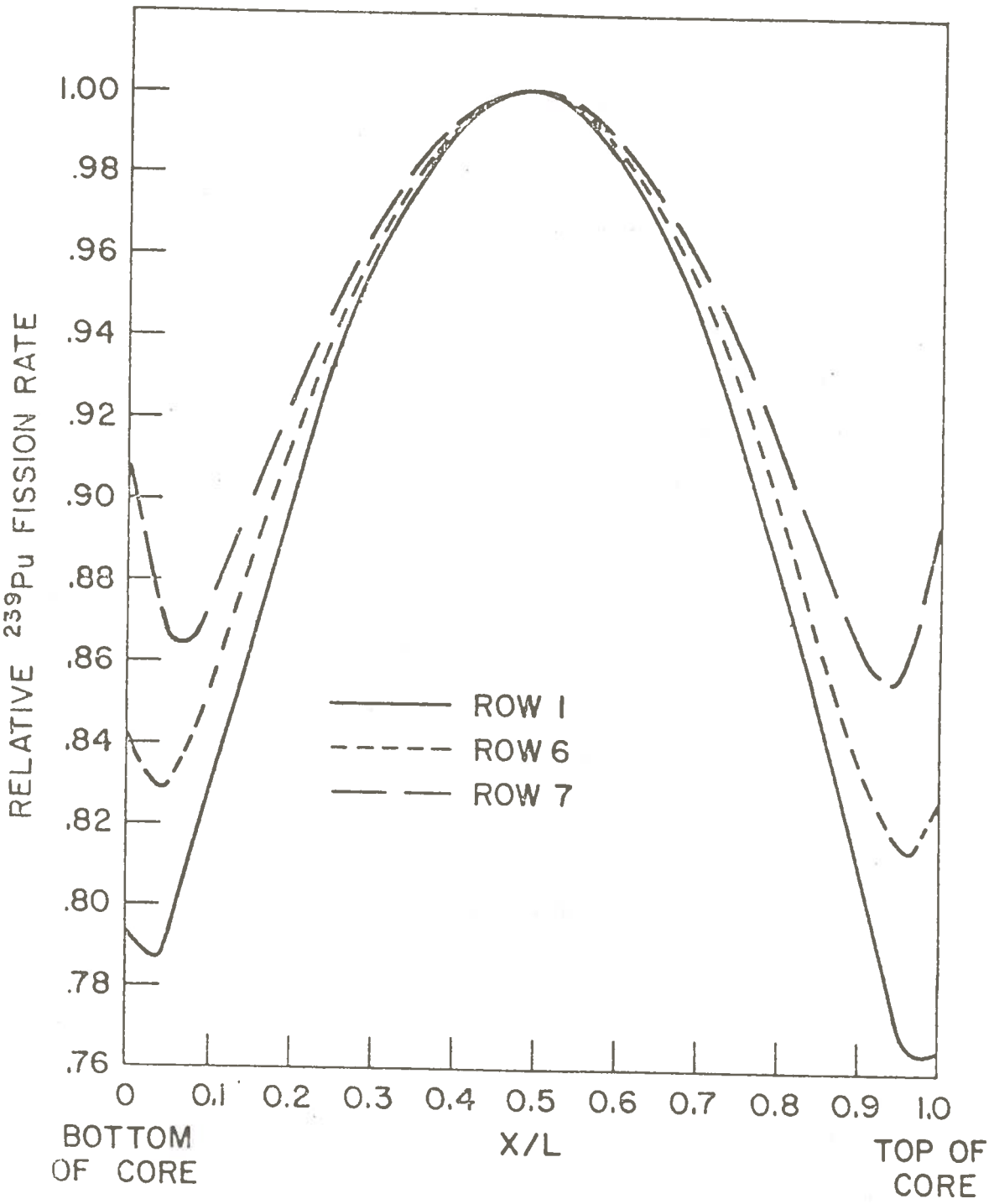

Figure I-4. Axial Distribution of ^{239}Pu Fission Rate in Driver Fuel in the EBR-II Core

perturbations, such as control rod drops and control rod oscillations. Techniques such as these which are applicable to this thesis are described in detail later. Analytically, use is made of various diffusion theory and transport theory computer codes. These various analyses are helpful in identifying reactor kinetic trends and help preclude undesired excursions in the reactor.

Statistical error may exist in the previously accepted values of feedback reactivity in use at EBR-II; this error could be significant in effect. There exist several potential contributors to this error, one being error in the drop rod worth. Error in the accepted number of delayed neutron groups and error in the weighting of the groups are other possible contributors, as are errors in the time constants which specify the rates of decay of the delayed neutron precursors. Terms with which the reader may be unfamiliar and which apply to this thesis are presented in a later discussion.

This thesis analyzes errors in the delayed neutron fractions related to possible errors in feedback reactivity, focusing on the effect of the individual delayed neutron fractions with short time constants on the prompt portion of feedback reactivity. Consideration is given to the effect of the total delayed neutron fraction on feedback reactivity. Finally, the derivative term of the reactor kinetics equations is examined analytically to determine its importance to fast reactor kinetics and its applicability to kinetics algorithms in use at EBR-II.

CHAPTER II

Reactor Kinetics and Inverse Kinetics

The reactor transient analyses conducted at EBR-II are considered to be among the more important studies performed in support of the LMFBR program. It is these analyses that reveal how the reactor system is affected by any changes in fuel loading or burnup, addition of experimental subassemblies, reactor start-up and shut-down, variation in power level, or any other changes in the operation of the reactor. Thus, kinetic trends are identifiable and can be contolled if necessary. The following reactor kinetics discussion is presented as a preliminary to specific kinetics applications at EBR-II.

An essential property of a fission neutron multiplying system is the infinite multiplication factor, k_∞. It is defined as the ratio of the number of neutrons in one generation to the number of neutrons in the next generation in a system of infinite size. For a system of finite size, some neutrons are lost through leakage. It is then appropriate to define the effective multiplication factor, k_e, as the ratio of the number of neutrons resulting from fission in each generation to the total number lost by both absorption and leakage in the preceeding generation. In order that a nuclear reaction be self-sustaining, which is a condition known as criticality, k_e must equal unity. If k_e is less than one, subcriticality exists and the reaction will expire. If k_e is

greater than one, the condition is supercriticality, and the chain reaction is divergent. A measure of the probability that neutrons will not leak out of a finite system, but will remain until absorbed, is called the nonleakage probability of the system. Therefore, k_e becomes the product of k_∞ and the nonleakage probability.

In specifying the infinite multiplication factor for a particular system, it is convenient to separate k_∞ into its contributing parts. For a thermal neutron system, $k_\infty = \varepsilon\eta fp$. This is the four factor formula, where

ε = the fast fission factor,

η = the thermal fission factor,

f = the thermal utilization factor,

p = the resonance escape probability.

More complete treatments of the four factor formula can be found in several standard texts.[4,5,9] A standard approach to fast reactor analysis considers only one energy group of neutrons. For a fast neutron system the four factors reduce to two factors, and k_∞ equals $f\eta$. In this case ε is inappropriate since fissions induced by neutrons of all energies are included in the term, η. The parameter, p, is not significant because few neutrons reach intermediate and thermal energies in a fast system. The factor, f, becomes a fuel utilization factor for absorption of neutrons of all energies in the nuclear fuel.

The neutron lifetime, ℓ, is the time elapsed between a neutron's birth through fission and its loss from the system by absorption or leakage. For a thermal system, ℓ includes the slowing down time and the diffusion time, while that portion of the lifetime that includes slowing down is insignificant in a fast system.

Delayed neutrons are neutrons produced through the decay of fission products. The fraction of neutrons in a system that are delayed neutrons, or neutrons which are not produced directly in the fission process itself, is β, the delayed neutron fraction. β is an experimentally obtained quantity, and is unique for a given nuclear fuel species. Since EBR-II is a multi-species device, an effective delayed neutron fraction is calculated based on the relative amounts of fissionable materials in a given core configuration. Usually associated with β are six delayed neutron groups, β_i, $i = 1,2,\ldots,6$, distinguishable by the difference in half-lives of the delayed neutron group precursors, C_i, $i = 1,2,\ldots,6$. It is the precursors in each group which are usually directly produced in fission. Table II-1 displays a typical set of some delayed neutron data applicable to EBR-II.[11] Delayed neutrons have the effect of increasing the neutron lifetime of a system. As such, the effective neutron lifetime, ℓ^*, which includes the effect of delayed neutrons, is given by $\ell^* = \ell + \beta\bar{\tau}$, where $\bar{\tau}$ is the average mean life of delayed neutrons in a system.

Table II-1. Delayed Neutron Data[11]

Group No. (i)	Half Life (sec)	Decay Constant (sec^{-1})	Relative Yield (β_i/β)
1	54.58	0.0127	3.4541×10^{-2}
2	21.87	0.0317	2.0402×10^{-1}
3	6.03	0.115	1.8510×10^{-1}
4	2.23	0.311	4.0232×10^{-1}
5	0.50	1.40	1.4089×10^{-1}
6	0.18	3.87	3.3133×10^{-2}

Increasing or decreasing the rate of reaction, and therefore the power level, of a reactor is accomplished by introducing either positive or negative reactivity to the system. Reactivity, or ρ, is a measure of the relative departure of a neutron system from the critical condition. The excess multiplication factor, δk_e, is obtained by subtracting one, the value of k_e for a critical system, from k_e at some time. Then reactivity equals δk_e divided by k_e.

The time required to increase the neutron population of a reactor by e, the base of the natural logarithms, is the reactor period, or T. In the absence of delayed neutrons, T equals ℓ divided by δk_e, so that the smaller the neutron lifetime, the smaller must be the departure from critical for a reactor with specific period. If delayed neutrons played no significant part in the maintenance of criticality of a system, the neutron lifetime would be so small as to make control of a fast reactor extremely difficult.

Reactivity can be expressed in either of two units of measurement. One is the dollar; by definition, one dollar is the amount of reactivity equal to the total delayed neutron fraction, or $\$ = \rho/\beta$. If one dollar of reactivity is present then a reactor is said to be prompt critical, which means the delayed neutrons are insignificant in effect. Similarly, one cent of reactivity equals one percent of a dollar. The other reactivity unit is the inhour, which is the quantity of reactivity that will induce a period of one hour in a neutron multiplying system. Solution of the inhour equation with T = 3600 seconds yields a reactivity of one inhour:

$$\rho = \frac{\ell}{k_e T} + \sum_{i=1}^{6} \frac{\beta_i}{(1 + \lambda_i T)} \quad . \qquad \text{(II-1)}$$

Derivations of the inhour equation can be found in the References.[4,5] For EBR-II, one dollar equals approximately 301 inhours.

The basic reactor kinetics equations, excluding external source terms, are

$$\frac{dn(t)}{dt} = \frac{[\rho(t) - \beta]n(t)}{\ell} + \sum_{i=1}^{6} \lambda_i C_i(t) , \qquad \text{(II-2)}$$

and

$$\frac{dC_i(t)}{dt} = \frac{\beta_i n(t)}{\ell} - \lambda_i C_i(t), \; i = 1,2,\ldots,6 \quad . \qquad \text{(II-3)}$$

In these equations, reactor power is proportional to the time-dependent neutron population, n(t). Essentially, power is evaluated given an applied reactivity.

Inverse kinetics analysis, on the other hand, evaluates applied reactivity given the power history of the reactor. If Equation (II-2) is solved for $\rho(t)$, the equation becomes

$$\rho(t) = \frac{\ell}{n(t)} \frac{dn}{dt} + \beta - \frac{\ell}{n(t)} \sum_{i=1}^{6} \lambda_i C_i(t). \qquad \text{(II-4)}$$

Appendix A includes a derivation of the specific inverse kinetic equations used in analyses at EBR-II.

The rod-drop experimental technique is a useful tool of kinetics research at EBR-II. Descriptions of this technique are well documented.[10] The drop rod is stainless steel and is arranged so that its release will allow it to drop out of the reactor core; the subsequent void then fills with liquid sodium. At predominant neutron

energies in the EBR-II core, the scattering properties of the rod are such that the reactor system experiences a net reactivity loss when the rod drops from the core.

Basically, feedback reactivity is arrived at using rod-drop data in the following manner: first, the drop rod is released at a reactor thermal power level of about 500 kWt, feedback effects being negligible at this low power. About 2.5 cents of negative reactivity is introduced by the drop rod release, and as the reactor temperature decreases, density increases occur within the reactor core. From the resulting power trace data are collected, and the power data are converted to reactivity without feedback by inverse kinetics procedures. Then, at a higher power level, the drop rod is released and reactivity is again determined in the above manner. At this higher power, the temperature-induced density changes are significant enough to feed reactivity back into the reactor. This reactivity at high power is subtracted point-by-point from that at 500 kWt, with the result being the feedback reactivity. Figure II-1 graphically illustrates the technique.

The EBR-II on-line Data Acquisition System (DAS), employing a Xerox Data Systems Sigma-5 Computer, is used for reduction of data. Digital computer codes used to process the power and reactivity data from the rod-drop experiments are the rod-drop calibration code, RCL, the feedback-reactivity calculation code, FBCK, and the power and feedback-reactivity plotting code, RPL.[1,8,11] The RCL code requires input in the form of control cards and a raw-data tape of the power

data at very low power; conversion of the data to position-dependent reactivity is done by inverse-kinetics procedures. The position-dependent reactivity data then provide a calibration basis (see Figure II-2) for FBCK, which provides reactivity data for rod drops at low power and feedback reactivity at higher power. FBCK also produces six-point-averaged feedback and power data for plotting by the RPL code (see Figure II-1).

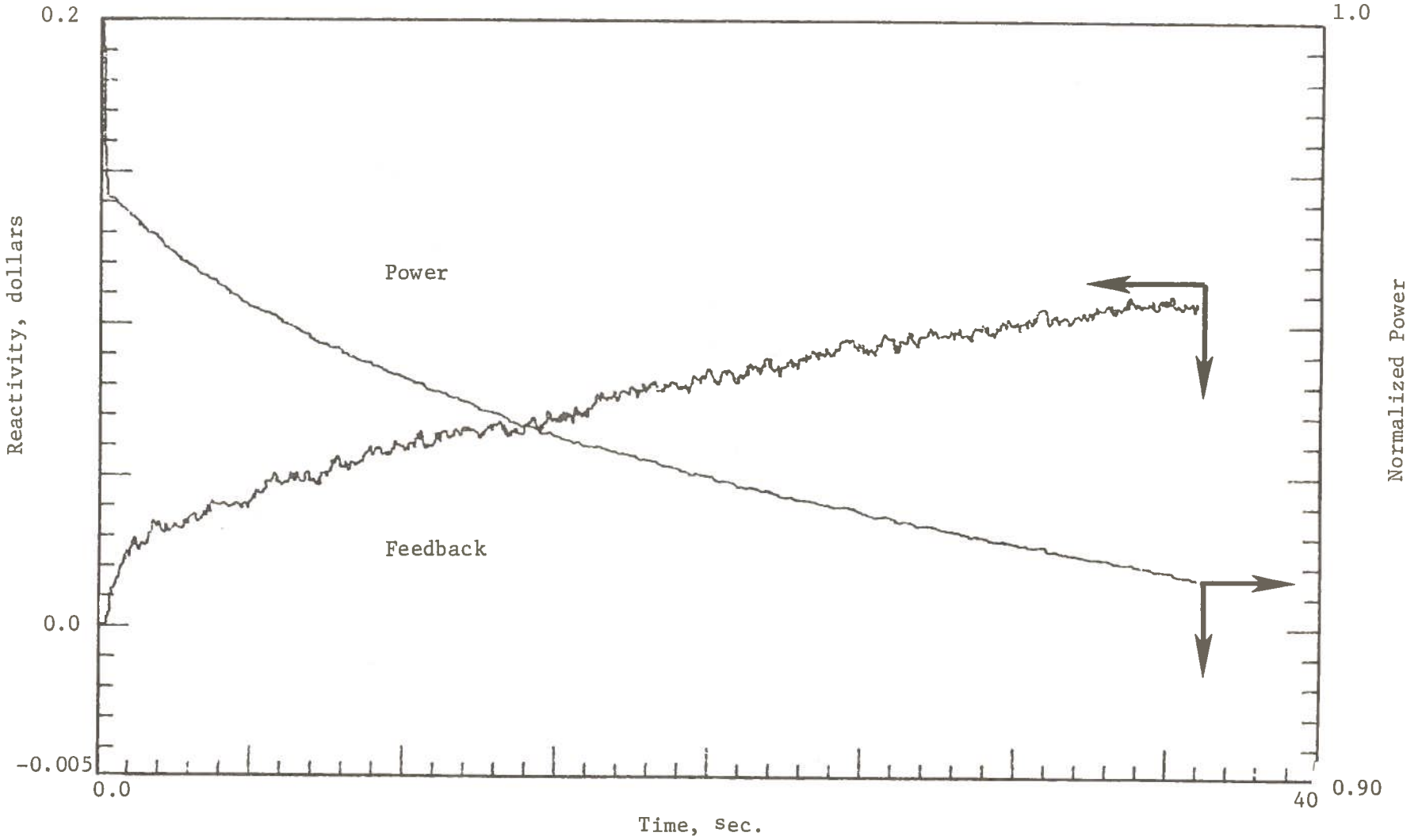

Figure II-1. Plot of Reactivity and Power vs Time, the Reactivity Plotting Code Output

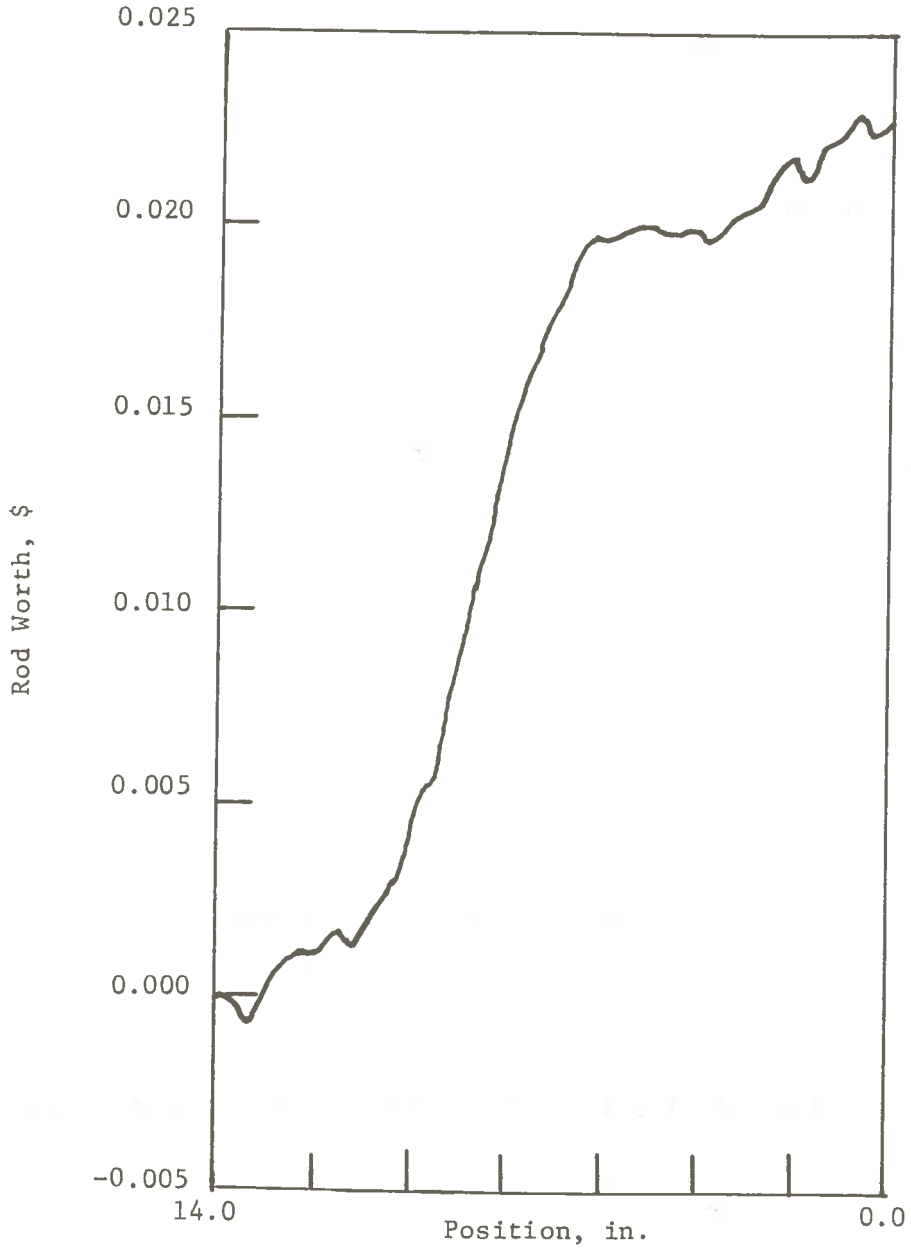

Figure II-2. Typical Rod Drop Calibration Curve

CHAPTER III

Experimental Techniques and Results

Measurements of the reactor system transfer function have provided the basis for one of the more informative, accurate and reliable methods used to assess the stability characteristics of a reactor. An input function of a system is a mathematical representation of a system input, such as reactivity insertion; similarly, an output function of a system is a mathematical representation of an output of the system, such as reactor power. A system transfer function is the ratio of the Laplace transform of the output function to that of the input function, and describes a system's characteristic behavior. This description permits the development of a mathematical model which can be used to determine reactor-system stability and the control system characteristics necessary for automatic control.[10]

Feedback reactivity can be expressed as,

$$\rho_{FB}(t) = \int_{-\infty}^{t} h(t-\tau)P(\tau)d\tau \ , \qquad \text{(III-1)}$$

a convolution integral, where h(t) is the feedback function, and P(t) is the relative power change. This change can be given as

$$\frac{p(t) - p_o}{p_o} \ ,$$

with p_o being the initial power level.

If p(t) equals p_o, an equilibrium power level, prior to t = 0, then

$$P(t) = \frac{p(t) - p_o}{p_o} = 0, \text{ and}$$

$$\rho_{FB}(t) = \int_{-\infty}^{o} h(t-\tau)P(\tau)d\tau + \int_{o}^{t} h(t-\tau)P(\tau)d\tau$$

$$= \int_{o}^{t} h(t-\tau)P(\tau)d\tau \qquad \text{(III-2)}$$

or, in Laplace transform notation $R_{FB}(s) = H(s)P(s)$.

Equation (III-1) can be written as

$$\rho_{FB}(t) = \int_{o}^{\infty} h(\lambda)P(t-\lambda)d\lambda \ , \qquad \text{(III-3)}$$

$$\text{so} \quad \rho_{FB}(\infty) = P(\infty) \int_{o}^{\infty} h(\lambda)d\lambda$$

$$\text{or } \rho_{FB}(\infty) = P(\infty)H(0). \qquad \text{(III-4)}$$

Similarly, reactor power can be expressed as

$$P(t) = \int_{o}^{t} g(t-u)\rho(u)du,$$

where g(t) is the reactor input function and $\rho(u)$ is the applied reactivity. The transform of the input function can be written as

$$G(s) = G_o(s)\ [1 + G_o(s)H(s)]^{-1} \ ,$$

$$\text{and since} \quad \lim_{s\to o} G_o(s) \to \infty, \ G(0) = \frac{1}{H(0)} \ , \qquad \text{(III-5)}$$

or the reactor transfer function, evaluated at s = 0, is the reciprocal of the feedback transfer function evaluated at s = 0.

For a step reactivity insertion

$$\rho(u) = \Delta\rho_o \ ,$$

$$P(t) = \Delta\rho_o \int_o^t g(t-u)du \qquad \text{(III-6)}$$

and $$P(\infty) = \Delta\rho_o G(0) = \frac{\Delta\rho_o}{H(0)} \ . \qquad \text{(III-7)}$$

Thus $$\rho_{FB}(\infty) = P(\infty)H(0) = \frac{\Delta\rho_o}{H(0)}\,[H(0)], \qquad \text{(III-8)}$$

or the final feedback reactivity value equals the inserted reactivity. And, since

$$P(\infty) = \frac{p(\infty) - p_o}{p_o} = \frac{\Delta p}{p_o},$$

$$\frac{\Delta\rho_o}{\Delta p} = \frac{H(0)}{p_o} \ . \qquad \text{(III-9)}$$

This defines the power coefficient, and states that the change in reactivity with respect to a change in power is equal to the feedback transfer function evaluated at zero divided by the equilibrium power level.[10]

Measured feedback reactivity data are fit to a frequency-dependent, linear feedback transfer function the form of which is empirically known.[10] It is

$$H(s) = \sum_{i=1}^{I} \frac{A_i e^{-sT_i}}{1 + s\tau_i} \qquad \text{(III-10)}$$

where τ_i is the time constant for the term, A_i is the magnitude of that term, the delay term is given by T_i, and H(s) is the Laplace transform of feedback function h(t).

For the prompt feedback reactivity, two indicators are associated with errors in the delayed neutron fractions with short time constants. One indicator of effect is the prompt power coefficient, or PPC, and the other indicator is prompt feedback reactivity, itself.

Delayed neutron group data for the β_i's with the three shortest half-lives were varied by specific amounts and the effects on prompt feedback reactivity and prompt power coefficient are noted. The basis for comparison is the delayed neutron data currently accepted for EBR-II and displayed in Table II-1. Errors were introduced in the delayed neutron groups by increasing each group fraction and combination of group fractions (groups four through six) by specific amounts. The total delayed neutron fraction, β, was subsequently increased, as were the normalized delayed neutron fractions, β_i/β. The new data were then used as input to the rod drop calibration code, RCL,[1,8,11] and the feedback reactivity calculation code, FBCK.[1,8,11] Effects were seen as, generally, decreases in the values of prompt feedback reactivity. The specific variations in delayed neutron data and their respective effects on prompt feedback reactivity are given in Table III-1. It can be seen that the maximum error in prompt feedback reactivity is induced by a twenty percent variation in delayed neutron groups four through six; this error is nineteen percent. A graphical representation of the effect on prompt feedback reactivity of errors in the delayed neutron fractions is included in Figure III-1.

Table III-1. Errors Induced in Feedback Reactivity

Run	Maximum % Error in Feedback Reactivity
orig. β's	--
β_4 10%$^+$	4.2
" 15%$^+$	4.2
" 20%$^+$	8.7
β_5 10%$^+$	8.7
" 15%$^+$	8.7
" 20%$^+$	13.6
β_6 15%$^+$	4.2
" 20%$^+$	4.2
β_4,β_5 10%$^+$	8.7
" 15%$^+$	13.6
" 20%$^+$	19.0
β_4,β_6 10%$^+$	4.2
" 15%$^+$	8.7
" 20%$^+$	8.7
β_5,β_6 10%$^+$	8.7
" 15%$^+$	13.6
" 20%$^+$	13.6
β_4,β_5,β_6 10%$^+$	8.7
" 15%$^+$	13.6
" 20%$^+$	19.0

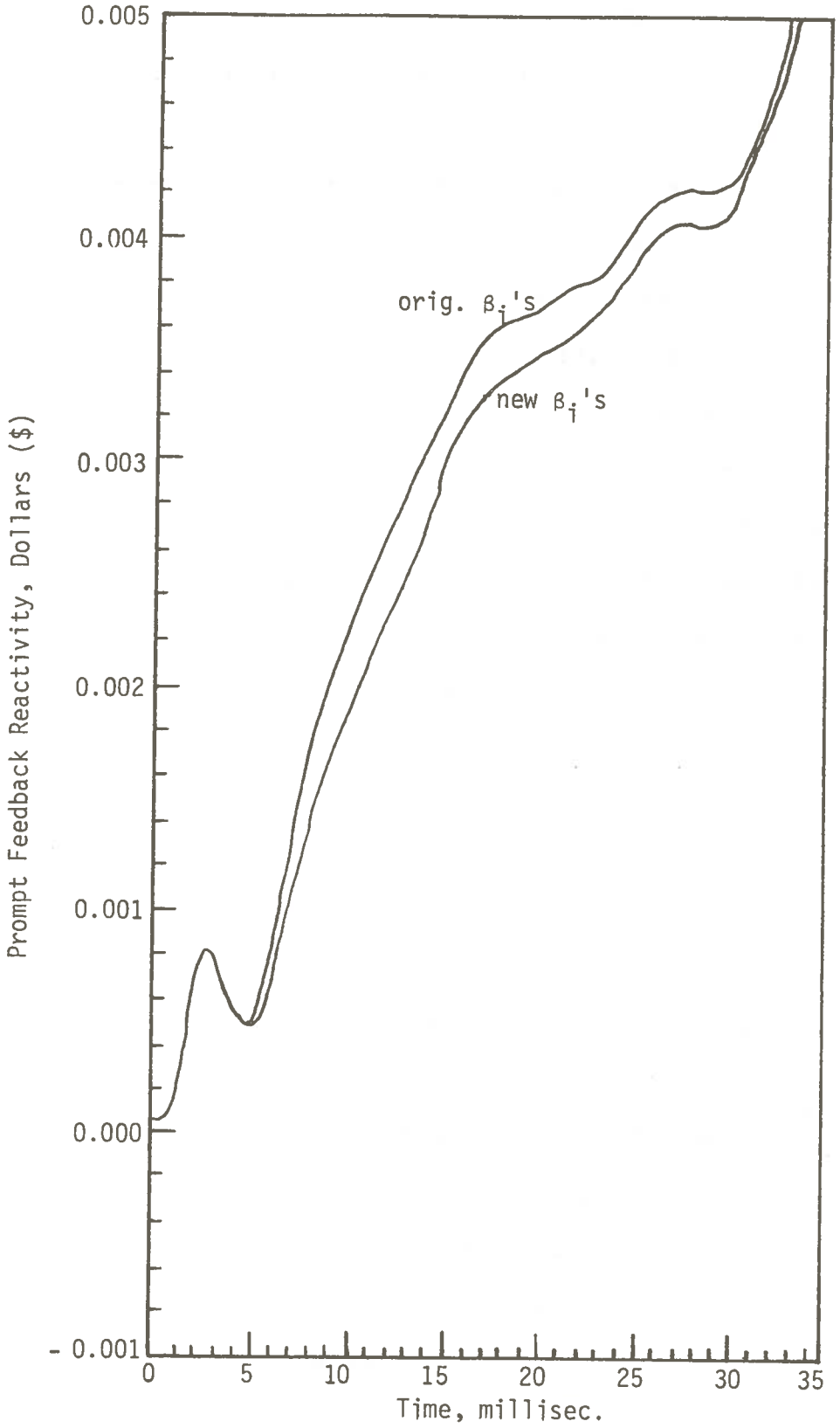

Figure III-1. Typical Prompt Feedback Reactivity Comparison Curve

The prompt power coefficient is obtained from the prompt feedback reactivity data by a variable metric minimization least squares data fitting technique. A complete description of this process can be found in the References.[3] Essentially, this method provides a means for determining numerically minima, on a local basis, of differentiable functions of several variables. In the minima determination process, a matrix is found which describes characteristically the behavior of functions about the minima. For a region in which the function exhibits a quadratic dependence on the variables, the maximum number of iterations required is equal to the number of variables. Variables can be linearly constrained by careful choice of starting values. The computer technique by Herman[6] requires input in the form of cards from the FBCK [1,8,11] code; the PPC data appear as output from this code.

Table III-2 presents the resulting errors in prompt power coefficient induced by variations in the delayed neutron parameters. The prompt power coefficient is given in Inhours per Megawatt, Ih/MW. The maximum error is 6.43 percent, resulting from a twenty percent variation in delayed neutron groups four through six. Additionally, applicability of the PPC data to graphical display was evaluated through a least-squares-fitting computer routine.[12] This code performs a two parameter (one parameter fixed) linear fit of prompt power coefficient vs delayed neutron group variation data. The fitting function used by this routine is

$$y(x) = \sum_{i=1}^{N} a_i x^{i-1} \; ; \qquad \text{(III-11)}$$

Table III-2. Errors Induced in Prompt Power Coefficient

Run	Prompt Power Coeff. (Ih/MW)	% Error in PPC
orig. β's	0.77003	--
*β_4 10%$^+$	0.76250	0.99
" 15%$^+$	0.75902	1.45
" 20%$^+$	0.75468	2.03
β_5 10%$^+$	0.75590	1.87
" 15%$^+$	0.74679	3.11
" 20%$^+$	0.74033	4.01
β_6 15%$^+$	0.76561	0.58
" 20%$^+$	0.76424	0.76
β_4,β_5 10%$^+$	0.74707	3.07
" 15%$^+$	0.73828	4.30
" 20%$^+$	0.72815	5.75
β_4,β_6 10%$^+$	0.76006	1.31
" 15%$^+$	0.75389	2.14
" 20%$^+$	0.74804	2.94
β_5,β_6 10%$^+$	0.75084	2.56
" 15%$^+$	0.74345	3.58
" 20%$^+$	0.73674	4.52
β_4,β_5,β_6 10%$^+$	0.74510	3.35
" 15%$^+$	0.73383	4.93
" 20%$^+$	0.72352	6.43

*The %$^+$ indicates a positive change of given percentage in the given delayed neutron fraction or combination of fractions.

the first two terms of the sum are $a_1 + a_2 x$. Here, a_1 represents the axis-intercept and a_2 represents the slope of the line fit to the data points. The variable y represents the prompt power coefficient value and the variable x represents variations in the given delayed neutron group or combination of groups. Table III-3 displays the results of this fit; values of standard deviation are approximately two orders of magnitude smaller than values of the slope of the line.

Variation of the total delayed neutron fraction is examined in an analytical manner. Let ε equal the amount by which the total delayed neutron fraction, β, is varied, and let ε' equal the amount by which each of the individual delayed neutron fractions, β_i, are varied. The FBCK[1,8,11] code calculates normalized delayed neutron fractions, β_i/β, to be used in the evaluation of feedback reactivity. In this case, the normalized delayed neutron fractions are

$$\frac{\beta_i + \varepsilon'\beta_i}{\beta + \varepsilon\beta} \quad .$$

Since $$\beta = \beta_1 + \beta_2 + \ldots + \beta_6 \quad , \tag{III-12}$$

$$\varepsilon\beta = \varepsilon'\beta_1 + \varepsilon'\beta_2 + \ldots + \varepsilon'\beta_6 \tag{III-13}$$

by our initial assumptions. It follows that

$$\varepsilon = \varepsilon' \; ; \tag{III-14}$$

then $$\frac{\beta_i + \varepsilon'\beta_i}{\beta + \varepsilon\beta} = \frac{\beta_i(1 + \varepsilon')}{\beta(1 + \varepsilon)} = \frac{\beta_i}{\beta} \tag{III-15}$$

Table III-3. Results of Least Squares PPC Data Fit

Delayed-Neutron Groups	a_2 (slope)	Standard Deviation of a_2
4*	7.55×10^{-2}	8.63×10^{-4}
5	1.50×10^{-1}	2.52×10^{-3}
6	2.51×10^{-2}	5.80×10^{-3}
4,5	2.13×10^{-1}	3.90×10^{-3}
4,6	1.08×10^{-1}	1.97×10^{-3}
5,6	1.73×10^{-1}	5.11×10^{-3}
4,5,6	2.38×10^{-1}	3.53×10^{-3}

* Group 4 represents variation in PPC vs variations in β_4 (10%, 15%, 20%). See Table III-2. The parameter a_1, which is the axis-intercept, was held fixed at zero.

This is the original normalized delayed neutron fraction, which implies that variations in β are canceled by equivalent variations in β_i/β when the kinetics derivative is ignored. This result was confirmed by values of prompt feedback reactivity calculated by FBCK.[1,8,11]

The derivative term of the kinetics equations is treated analytically to determine its applicability to fast reactor kinetics.[7] The kinetics equations are, neglecting source terms,

$$\frac{dn(t)}{dt} = \frac{\rho(t) - \beta}{\ell} n(t) + \sum_{i=1}^{6} \lambda_i C_i(t) \qquad \text{(III-16)}$$

and $$\frac{dC_i(t)}{dt} = \frac{\beta_i n(t)}{\ell} - \lambda_i C_i(t), \quad i = 1, 2, \ldots, 6. \qquad \text{(III-17)}$$

The neutron lifetime, ℓ, is small (about 10^{-7} sec.) for fast reactors such as EBR-II. If ρ is much less than β, then the right-hand side of equation (III-16) contains a large negative term $(\rho - \beta)n/\ell$. As such, the derivative dn/dt becomes the possibly small difference between the large negative term and a large positive term

$$\sum_{i=1}^{6} \lambda_i C_i \ .$$

If the neutron density, n, is expanded in powers of the small parameter, ℓ, a technique known as the method of singular perturbations,[7]

$$n = n_1 \ell^{\circ} + n_2 \ell + \ldots \qquad \text{(III-18)}$$

and $$\frac{dn}{dt} = \frac{dn_1}{dt} + \frac{\ell dn_2}{dt} + \ldots \qquad \text{(III-19)}$$

Time dependence of the quantities n, ρ, and C_i is understood. Considering the first two terms of the sum, assuming the time derivatives are small, and substituting into Equation (III-16),

$$\frac{dn_1}{dt} + \frac{\ell dn_2}{dt} = \frac{\rho - \beta}{\ell} n_1 + (\rho - \beta) n_2 + \sum \lambda_i C_i \quad . \tag{III-20}$$

Equating terms of order ℓ^{-1},

$$0 = \frac{\rho - \beta}{\ell} n_1 + \sum \lambda_i C_i \quad , \tag{III-21}$$

since in equilibrium, from Equation (III-17),

$$\lambda_i C_i = \frac{\beta_i n_o}{\ell} \quad . \tag{III-22}$$

Solution of Equation (III-21) for n_1 produces

$$n_1 = \frac{\sum \ell \lambda_i C_i}{\beta - \rho} \quad . \tag{III-23}$$

Similarly, equating terms of order ℓ°,

$$\frac{dn_1}{dt} = (\rho - \beta) n_2 \quad , \tag{III-24}$$

or

$$n_2 = \frac{dn_1}{dt} \frac{1}{\rho - \beta} \quad . \tag{III-25}$$

Differentiation of Equation (III-23) and substitution into the above equation yields

$$n_2 = \frac{-1}{(\beta - \rho)^2} \left(\sum \lambda_i \ell \frac{dC_i}{dt} + \frac{\sum \lambda_i \ell C_i}{\beta - \rho} \frac{d\rho}{dt} \right) \quad . \tag{III-26}$$

The neutron density, n, is now the sum of Equations (III-23) and (III-26).

If only one group of delayed neutrons is considered, the equation for the neutron density is

$$n = \frac{\lambda \ell C}{\beta-\rho} - \frac{\ell}{(\beta-\rho)^2}\left(\lambda \ell \frac{dc}{dt} + \frac{\lambda \ell C}{\beta-\rho}\frac{d\rho}{dt}\right). \qquad \text{(III-27)}$$

Using the expression for dC/dt given by Equation (III-17),

$$n = \frac{\lambda \ell C}{\beta-\rho} - \frac{\ell}{(\beta-\rho)^2}\left(\beta\lambda n - \lambda^2 \ell C + \frac{\lambda \ell C}{\beta-\rho}\frac{d\rho}{dt}\right). \qquad \text{(III-28)}$$

After factoring and collecting terms,

$$n = \frac{\lambda \ell C}{\beta-\rho}\left(1 - \frac{\ell}{(\beta-\rho)^2}\left(\lambda\rho + \frac{d\rho}{dt}\right)\right). \qquad \text{(III-29)}$$

Recall from Equation (III-23) that $\frac{\sum \ell \lambda_i C_i}{\beta-\rho}$ was valid for a small dn/dt; hence the term

$$\frac{\ell}{(\beta-\rho)^2}\left(\lambda\rho + \frac{d\rho}{dt}\right)$$

provides a correction on the "smallness" of dn/dt. The following condition should be met in order that the derivative term be insignificant:

$$\frac{\ell}{(\beta-\rho)}\left(\lambda\rho + \frac{d\rho}{dt}\right) << 1. \qquad \text{(III-30)}$$

The Inequality (III-30) is valid for values of reactivity approaching, but not equal to, the total delayed neutron fraction. The condition when ρ is greater than β, known as super prompt critical, is not considered here. Only pulse-type reactors operate at super prompt critical; these reactors are limited in application and do not simulate power reactor conditions.

CHAPTER IV

Conclusions and Recommendations

The major conclusions of this study are as follows:

(1) Prompt feedback reactivity is relatively sensitive to possible errors in the delayed neutron fractions four through six.

(2) Measurement of the prompt power coefficient (PPC) is a good means of assessing the sensitivity of reactivity to changes in reactor power. This coefficient is not as sensitive to possible errors in the delayed neutron parameters as is the prompt feedback reactivity.

(3) Neither the PPC nor the prompt feedback reactivity is affected by possible error in the total delayed neutron fraction.

(4) For fast reactors such as EBR-II, with short neutron lifetimes, the kinetics derivative term can be ignored in almost all normal applications. This conclusion does not, of course, hold for thermal reactors which have much longer neutron lifetimes.

Because of uncertainties in the delayed neutron parameters, the possibility of significant error in prompt feedback reactivity exists at EBR-II. This study has shown that a twenty percent variation in delayed neutron fractions four through six results in a maximum of

nineteen percent error in the prompt feedback reactivity. In addition, prompt feedback reactivity is essentially unaffected by errors in the total delayed neutron fraction; this conclusion was reached analytically and experimentally.

Due to the nature of the methods used to derive the prompt power coefficient from the rod-drop data, this coefficient is not as sensitive as is prompt feedback reactivity to errors in the delayed neutron fractions. A twenty percent error in the delayed neutron groups with the three shortest time constants produces an error of 6.43 percent in the prompt power coefficient. The accuracy follows from the fact that the computer codes used to perform the analyses very closely model the dynamic behavior of the reactor. The codes have been developed over a number of years and are in agreement with experimental data taken for a variety of reactor conditions.

For fast reactors with small neutron lifetimes, the kinetics derivative term, dn/dt, can be neglected under a variety of reactor conditions. When the derivative term is ignored, the validity of kinetics calculations is not affected in most instances. If the rate of reactivity change is as high as ten dollars per second (which is an extreme accident condition for EBR-II) then the conditions for neglecting dn/dt will be met even if ρ is as much as ninety five percent of prompt critical. If the rate of reactivity change is 2.5 cents per 120 milliseconds (which is realistic for EBR-II) then ρ can approach ninety eight percent of prompt critical. Appendix B

presents detailed values of reactivity and reactor conditions which allow the use of the simplified (no derivative term) kinetics expression.

The inequality (III-30) holds for one group of delayed neutrons. While it is true that one delayed neutron group is not as accurate as six groups, the conditions stated in Appendix B are useful for predicting the maximum permissible reactivity that allows the neglect of dn/dt. An extremely accurate quantitative prediction is possible if the more exact six group model is used in the analysis.

This thesis forms a basis for some continued investigations in fast reactor kinetics. Error analysis of the prompt power coefficient with respect to the time constants describing feedback reactivity at EBR-II is a specific topic which deserves further study. The effects of inherent reactor noise, and an inverse kinetic analytical prediction of errors in feedback reactivity should play important roles in additional investigations.

REFERENCES

1. J. M. Allen, et al., "Functional Description of the EBR-II Digital Data Acquisition System", ANL-EBR-02a (November 1970).
2. J. W. Allen, J. C. Robinson, and N. J. Ackerman, Jr., "Sensitivity Analysis of the Inverse Kinetics Rod-Drop Technique", TANSAO 17, 1-578 (November 1973).
3. W. C. Davidon, "Variable Metric Method for Minimization", ANL-5990 (May 1959).
4. A. R. Foster and R. L. Wright, Jr., Basic Nuclear Engineering, Allyn and Bacon, Inc., (1973).
5. S. Glasstone and A. Sesonske, Nuclear Reactor Engineering, Van Nostrand Reinhold Company (1967).
6. J. Herman, "Variable Metric Minimization", ANL-Z013S (April 1967).
7. D. L. Hetrick, Dynamics of Nuclear Reactors, The University of Chicago Press (1971).
8. R. W. Hyndman, et al., "Description of Software for the EBR-II Digital Data Acquisition System", ANL-EBR-033 (December 1970).
9. J. R. Lamarsh, Introduction to Nuclear Reactor Theory, Addison-Wesley Publishing Company, Inc., (1966).
10. H. A. Larson, "Kinetics Research and Testing at EBR-II", ANL-8086, (April 1974).

11. H. A. Larson, et al., "Computer Programs for Describing the Dynamic Behavior of EBR-II", ANL/EBR-076 (September 1973).

12. R. H. Moore and R. K. Zeigler, "The Solution of the General Least Squares Problem with Special Reference to High-Speed Computers", LA-2367 (October 1959).

APPENDIX A

Mathematical Derivation for Inverse-Kinetics Digital Computer Codes

The basic kinetics equations provide a means for evaluation of reactor power when the applied reactivity is known. One form of the kinetics equations is

$$\frac{dn(t)}{dt} = \left(\frac{k_{ex}(t)\ (1 - \beta) - \beta}{\ell}\right) n(t) + \sum_{i=1}^{6} \lambda_i C_i^*(t), \qquad \text{(A-1)}$$

$$\frac{dC_i^*(t)}{dt} = \left(\frac{k_{ex}(t) + 1}{\ell}\right) \beta a_i n(t) - \lambda_i C_i^*(t), \quad i = 1, 2, \ldots, 6. \qquad \text{(A-2)}$$

In order that applied reactivity be evaluated given the power history, Equations (A-1) and (A-2) must be modified. Particular inverse kinetics equations used in the FBCK[1,8,11] code are presented as follows.

Initially,

$$C_i^*(0) = \frac{\beta a_i}{\lambda_i}\ n_o \,. \qquad \text{(A-3)}$$

If $$C_i(t) = C_i^*(t) - C_i^*(0)$$

$$= C_i^*(t) - C_{io}^*,$$

then $$C_i(0) = 0.$$

Substituting Equation (A-3) into (A-2),

$$\frac{dC_i(t)}{dt} = \left(\frac{k_{ex}(t) + 1}{\ell}\right) \beta a_i n(t) - \lambda_i C_i(t) - \frac{\beta a_i n_o}{\ell} \,. \qquad \text{(A-4)}$$

Using two-point trapezoidal integration from t_o to t, and evaluation at $t = t + h$,

$$C_i(t + h) = e^{-\lambda_i h}\left(C_i(t) + \frac{\beta a_i h}{2\ell}\left([k_{ex}(t) + 1]n(t) - n_o\right)\right) + \frac{\beta a_i h}{2\ell}\left([k_{ex}(t + h) + 1]n(t + h) - n_o\right) \qquad (A\text{-}5)$$

Application of Equation (A-5) to Equation (A-1) now yields

$$\frac{dn(t)}{dt} = \frac{k_{ex}(t)(1 - \beta) - \beta}{\ell} n(t) + \frac{\beta n_o}{\ell} + \sum_{i=1}^{6} \lambda_i C_i(t)$$

$$= \frac{-\beta}{\ell}[k_{ex}(t) + 1]n(t) + \frac{\beta n_o}{\ell} + \frac{k_{ex}(t)n(t)}{\ell}$$

$$+ \sum_{i=1}^{6} \lambda_i C_i(t). \qquad (A\text{-}6)$$

Now, the following equation is produced by applying a three-point derivative formula to Equation (A-6):

$$\frac{n(t + 2h) - n(t)}{2h} = \frac{k_{ex}(t + h)}{\ell} n(t + h)\left(1 - \beta + \frac{h}{2}\sum_{i=1}^{6} \lambda_i \beta a_i\right)$$

$$+ \frac{[n(t + h) - n_o]}{\ell}\left(1 - \beta + \frac{h}{2}\sum_{i=1}^{6} \lambda_i \beta a_i\right)$$

$$- \frac{n(t + h) - n_o}{\ell} + \sum_{i=1}^{6} \lambda_i e^{-\lambda_i h}$$

$$X\left(C_i(t) + \frac{\beta a_i h}{2\ell}\left([k_{ex}(t) + 1]n(t) - n_o\right)\right).$$

Define $\Phi^{-1} = \left(1 - \beta(1 - \frac{h}{2} \sum_{i=1}^{6} \lambda_i a_i)\right)$, multiply through by ℓ and collect terms:

$$k_{ex}(t + h)n(t + h)\ \Phi^{-1} = - [n(t + h) - n_o]\ [\Phi^{-1} - 1] + (\ell)\frac{n(t + 2h) - n(t)}{2h}$$

$$- \sum_{i=1}^{6} \lambda_i e^{-\lambda_i h} \cdot \left(\ell C_i(t) + \frac{\beta a_i h}{2}\ \left([k_{ex}(t) + 1]n(t) - n_o\right)\right).$$

Divide by $n(t + h)$ and multiply by Φ:

$$k_{ex}(t + h) = \Phi\left[\frac{(\ell)n(t + 2h) - n(t)}{2h} - \sum_{i=1}^{6} \lambda_i e^{-\lambda_i h}\ [\ell C_i(t)}{n(t + h)} \quad \ldots$$

$$\ldots\ + \frac{(\beta a_i h/2)([k_{ex}(t) + 1]n(t) - n_o)}{n(t + h)}\right] + \frac{n(t + h) - n_o}{n(t + h)}\ (\Phi - 1) \quad \text{(A-7)}$$

Equations (A-5) and (A-7) are programmed in the XDS Sigma-5 version of the inverse kinetics code FBCK.[1,8,11]

APPENDIX B

Kinetics Derivative Applications

In order that the kinetics derivative term, dn/dt, be small enough as to be neglected, the Inequality (III-30) should be satisfied. It is

$$\frac{\ell}{(\beta-\rho)^2}\left(\lambda\rho + \frac{d\rho}{dt}\right) << 1, \qquad \text{(B-1)}$$

where $\rho \rightarrow \beta$. This may be rewritten as

$$\ell\lambda\rho + \ell\frac{d\rho}{dt} << (\beta-\rho)^2 . \qquad \text{(B-2)}$$

For values of ρ and $d\rho/dt$ which are given below, satisfaction of (B-2) will be attempted.

Values which are appropriate for fast reactors, and which apply specifically to EBR-II, are $\ell = 1.1 \times 10^{-7}$ seconds, $\beta = 6.8 \times 10^{-3}$, and $\lambda = 0.479$ seconds^{-1}. If $d\rho/dt$ equals ten dollars per second, a condition which is very extreme for EBR-II, then the Inequality (B-2) becomes, for the given values of reactivity:

ρ, \$	Inequality (B-2)
0.20	$7.55 \times 10^{-9} << 2.96 \times 10^{-5}$
0.50	$7.66 \times 10^{-9} << 1.16 \times 10^{-5}$
0.80	$7.77 \times 10^{-9} << 1.85 \times 10^{-6}$
0.90	$7.80 \times 10^{-9} << 4.62 \times 10^{-7}$
0.95	$7.82 \times 10^{-9} << 1.16 \times 10^{-7}$

If $d\rho/dt$ equals 2.5 cents per 120 milliseconds, a condition which is appropriate for EBR-II, then the following is obtained:

ρ, \$	Inequality (B-2)
0.20	$2.29 \times 10^{-10} < < 2.96 \times 10^{-5}$
0.50	$3.36 \times 10^{-10} < < 1.16 \times 10^{-5}$
0.80	$4.44 \times 10^{-10} < < 1.85 \times 10^{-6}$
0.90	$4.80 \times 10^{-10} < < 4.62 \times 10^{-7}$
0.95	$4.98 \times 10^{-10} < < 1.16 \times 10^{-7}$
0.98	$5.08 \times 10^{-10} < < 1.85 \times 10^{-8}$
0.99	$5.12 \times 10^{-10} < < 4.6 \times 10^{-9}$

Evidently, for the two cases of rate of reactivity change, which are representative of both normal operation and extreme accident conditions, reactivity may be quite close to prompt critical and the Inequality (B-2) holds.

These values were obtained using a one delayed neutron group analysis. As such, the numbers should not be viewed as quantitatively exact, but should be regarded as useful rules of thumb.

VITA

Winton G. Aubert was born in Baton Rouge, Louisiana, on April 10, 1952. He was raised in Denham Springs, Louisiana, and was graduated in 1970 from Denham Springs High School.

He entered Louisiana State University in 1970, and received the Bachelor of Science degree in Engineering Science in 1974. He then began a Graduate program at Louisiana State University leading to the Master of Science degree in Nuclear Engineering, for which he is now a candidate.

EXAMINATION AND THESIS REPORT

Candidate: Winton George Aubert

Major Field: Nuclear Engineering

Title of Thesis: Error in Prompt Feedback Reactivity at EBR-II: A Parametric Study

Approved:

Major Professor and Chairman

Dean of the Graduate School

EXAMINING COMMITTEE:

Date of Examination:

November 24, 1975